EFFECTIVE DEFECT IDENTIFICATIONS IN HONEYCOMBS

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Summary The image reconstruction problem based on Electrical Impedance Tomography (EIT) is an ill-posed inverse problem of finding such conductivity distribution that minimizes some optimisation criterion, which can be given by a suitable primal objective function. This paper describes new algorithms for the reconstruction of the surface conductivity distribution, which are based on stochastic methods to be used for the acquirement of more accurate reconstruction results and stable solution. The proposed methods are expected to non-destructive test of materials. There are shown examples of the identification of voids or cracks in special structures called honeycombs. Instead of the experimental data we used the phantom evaluated voltage values based on the application of finite element method. The results obtained by this new approach are compared with results from the known deterministic approach to the same image reconstruction.

1. INTRODUCTION

In Electrical Impedance Tomography an approximation of the internal conductivity distribution is computed based on the knowledge of the voltages and currents on the surface of the body. We have studied the possibilities of using stochastic and deterministic methods to reconstruct static two-dimensional (2D) conductivity distribution on thin conductive layers of unknown surface conductivity and known geometry. An example of the so called honeycomb structure is shown in Fig. 1. There will be shown that the 2D static images obtained from the solution of inverse problem are able to recover the original values by means of suitable algorithms based on stochastic or deterministic methods. In inverse problems the forward problem is used to predict the observation. The frequency range of the applied current sources used in EIT is of the order of kHz. The corresponding wavelength of the electromagnetic wave is much larger than the dimension of the specimen under investigation so that curl electric field components as well as displacement current influence can be neglected and only the conductive currents are considered.

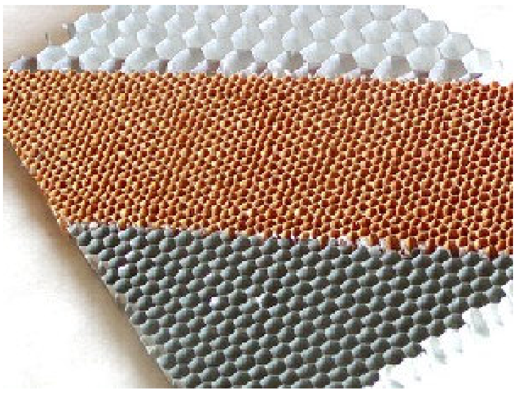

Fig. 1. An example of honeycombs

Further we assume the existence of the thin conductive layers only. Let grad_s and div_s be the surface gradient and the surface divergence operators on supposed conductive layer. This field is described by the continuity equation

$$\mathrm{div}_s\, \sigma_s\ \mathrm{grad}_s U = 0\ , \tag{1}$$

Here, U is the potential and σs is the unknown surface conductivity distribution in the phantom. The problem is solved as a static one. The solution of (1) satisfies the Dirichlet and Neumann boundary conditions, too. Equation (1) together with the complete electrode model [1] is discretized by the Finite Element Method (FEM). We approximated (1) from nodal values U_j using approximation functions N_j on a grid of linear triangular finite elements

$$U = \sum_{nodes} U_\mathrm{j}\ N_\mathrm{j}(x,y)\,. \tag{2}$$

Applying the Galerkin method to (1) and integrating by parts we have (**m** is the outer unit normal to the thin layer)

$$\int_{layer} \sigma_s \mathrm{grad}_s N_\mathrm{i} \cdot \mathrm{grad}_s U\, d\mathrm{l} - \int_c \sigma_s\, N_\mathrm{i}\, \mathrm{grad}_s U \cdot \boldsymbol{m}\, dc = 0. \tag{3}$$

The line integral along c is nonzero only for those nodes i that belong to curve c common to layer l and to the surface of the current supply electrodes. We include the electrode contact impedances according to [1] in (3) and we obtain the resulting discretized system of linear equations of the form

$$\boldsymbol{K}\,\boldsymbol{U} = \boldsymbol{F}\ . \tag{4}$$

2. PROPOSED RECONSTRUCTION TECHNIQUES

The EIT inverse problem searches for parameters in a high-dimensional space. Let propose the primal objective function

$$\Psi(\sigma_s) = \frac{1}{2}\sum \left\| \boldsymbol{U}_{\mathrm{M}} - \boldsymbol{U}_{\mathrm{FEM}}(\sigma_s) \right\|^2 . \tag{5}$$

Here σ_s is the surface conductivity distribution vector in the object, U_M is the vector of measured voltages on the boundary, and $U_{FEM}(\sigma_s)$ is the vector of computed peripheral voltages in respect to σ_s, which can be obtained using FEM. To minimize the objective function $\Psi(\sigma_s)$ we can use a lot of different methods based on both deterministic and stochastic approaches [2, 3]. When we apply the deterministic method based on the Least Squares (LS) method due to the ill-posed nature of the problem, regularization has to be used. It is possible to apply the widely known Tikhonov Regularization Method (TRM) or the Total Variation Primal Dual Interior Point Method (TV PD-IPM), described in [4, 5]. With respect to regularization the object function of TRM can be written in the form

$$\Psi(\sigma_s) = \frac{1}{2}\sum \left\| \boldsymbol{U}_{\mathrm{M}} - \boldsymbol{U}_{\mathrm{FEM}}(\sigma_s) \right\|^2 + \alpha \left\| L\sigma_s \right\|^2 \tag{6}$$

Here, α is the regularization parameter, and L is the so-called regularization matrix. The primal objective function $\Psi(\sigma_s)$ for TV PD-IPM algorithm

$$\Psi(\sigma_s) = \frac{1}{2}\sum \left\| \boldsymbol{U}_{\mathrm{M}} - \boldsymbol{U}_{\mathrm{FEM}}(\sigma_s) \right\|^2 + \alpha \sum \sqrt{\left\| \boldsymbol{L}\boldsymbol{\sigma}_{\mathrm{s}} \right\|^2 + \beta} \tag{7}$$

Here L is a suitable regularization matrix again and β is a small positive parameter, which represents an influence on the smoothing of $\Psi(\sigma_s)$. For the solutions of (6) and (7) we can apply a Newton-Raphson method. The iterative procedures are very sensitive to be trapped in local minima and so sophisticated regularization must be taken into account to obtain the stable solution.

A little bit different approach present global optimizing evolutionary algorithms, such as genetic algorithms, which have been recently applied to the EIT problem [6, 7]. Compared to the genetic algorithms, the Differential Evolution Algorithm (DEA) is a relatively new heuristic approach to minimizing nonlinear and non-differentiable functions in a real and continuous space. DEA can converge faster and with more certainty than many other global optimization methods according to various numerical experiments. It requires only a few control parameters and it is robust and simple in use. The details of the algorithm based on DEA can be found for example in [8]. Furthermore new algorithm so-called the Controlled Selection of Non-homogeneities (CSN) can be used to the reconstruction of the conductivity distribution [9]. The optimization of the primal objective function (5) based on the CSN is a relatively new technique with a very simple basic principle. All the briefly introduced methods were used to reconstruction of the surface conductivity distribution.

3. EXPERIMENTAL RESULTS

A simple example of 2D grid of the honeycomb structure is given in Fig. 2. The grid is fully described by its nodes and edges. The mesh for the calculation of the gradients, voltage reference values, and the Jacobians during iterations, has a total of 384 edges, and 272 nodes. The same finite element mesh is used for the forward and the inverse calculations. The volume conductivity is assumed to be zero. The surface conductivity σ_s has non-zero value 72 700 S (on black edges) except the black bold edges, where the actual value of conductivity σ_s has zero value and these edges represent some cracks in honeycombs structure, see the original conductivity distribution in the Fig. 2 and Fig. 4. We assume the constant distribution of the conductivity σ_s on all edges.

Two simple examples of some results of numerical experiments are presented in following figures. The number of current supply and voltage electrodes was 48 and the exciting currents were distributed trigonometrically with magnitude 1 mA. All the recovered values were obtained using modification of the TRM, PD-IPM, DEA and CSN algorithms. In the Fig. 3 and Fig. 5 are presented conductivity σ_s distributions on edges obtained using TRM, DEA and CSN algorithms. The conductivity distribution σ_s obtained using PD-IPM is not shown because it is very similar to results obtained using TRM, the accuracy of recovering results is a little bit worse in case of using PD-IPM.

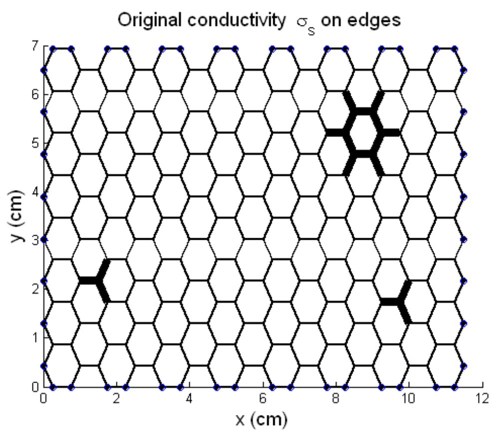

Fig. 2. Example 1, original distribution

When we use the TRM we needed 5 iterations to obtained results presented in Fig. 3, Fig. 5 and the accuracy in both cases is strong depending on the value of the regularization parameter α. We can see that the best results of the reconstruction were obtained always when we use the CSN algorithm.

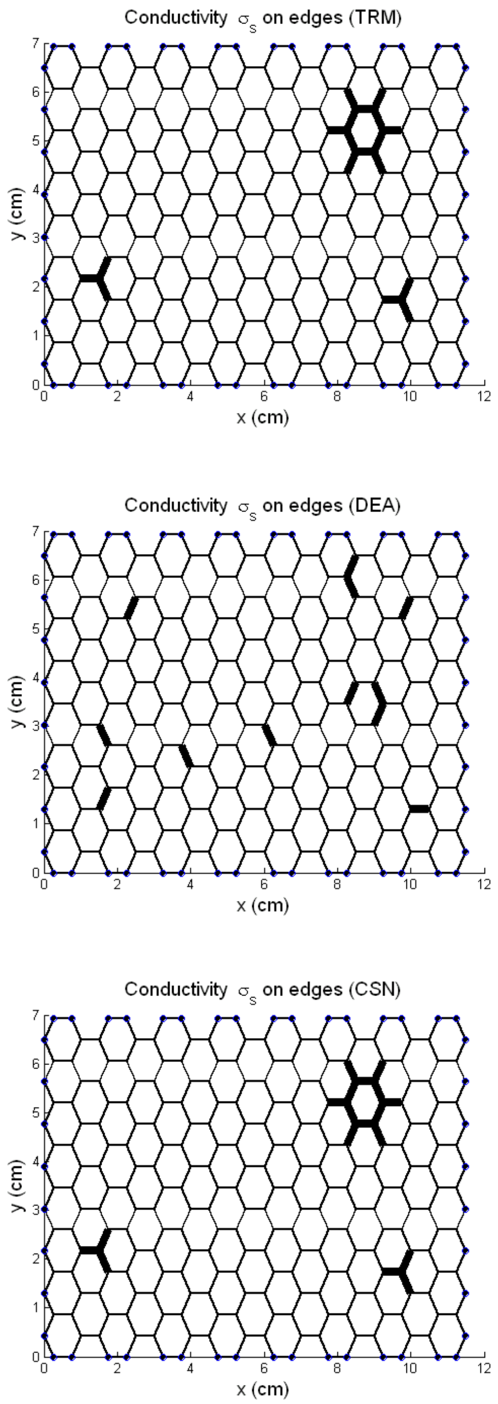

Fig. 3. Example 1, reconstruction results

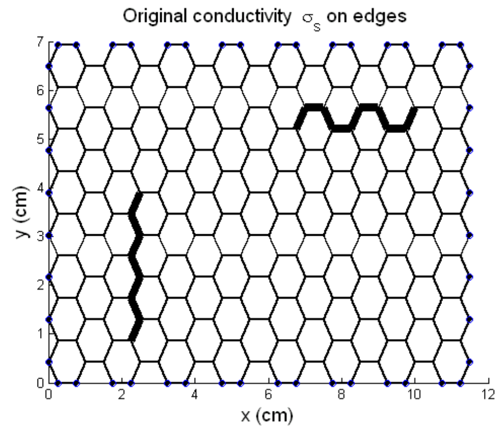

Fig. 4. Example 2, original distribution

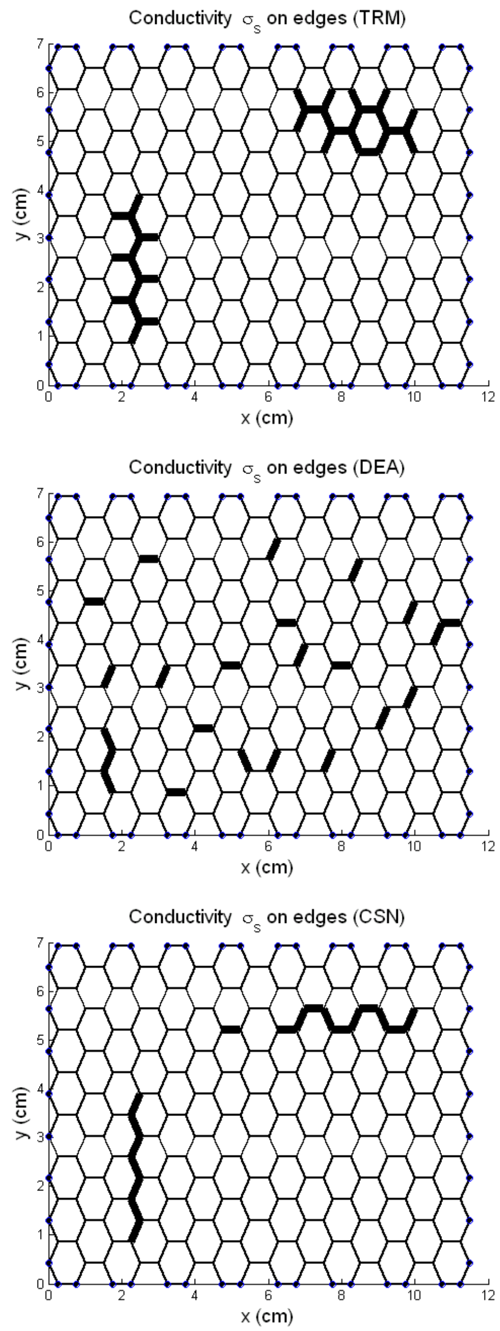

Fig. 5. Example 2, reconstruction results

4. CONCLUSION

In this paper the new possibilities to the reconstruction of non-homogeneities distribution has been presented. The all algorithms based on both deterministic and stochastic approaches have been adapted to reconstruction of conductivity distributions in special honeycomb structures. Based on many numerical experiments performed during the above-described algorithms and methods we can say that only the application of the CSN reconstruction algorithm has the significant advantage over the TRM in better accuracy and stability of the reconstruction process. On the other hand the CSN is very time-consuming technique. The stability of the TRM algorithm is a bit sensitive to the setting of the starting value of conductivity. The regularization parameter α controls the relative weighting allocated to the prior information. Its optimal choice provides balance between the accuracy and stability of the solution. On the basis of many numerical experiments, it is supposable that we obtain higher accuracy of the reconstruction results for smaller value of the parameter α, but if the value of α is decreasing, the instability of the solution is increasing. The results stated above as well as many other examples were obtained using a program written in MATLAB by author.

The next paper will present other possibilities and some examples to obtain the effective reconstruction results in more practical cases with respect to the best accuracy, stability and space resolution of non-homogeneities.

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REFERENCES

[1] Vilhunen, T., Kaipio, J., P., Savolainen, T. and Vauhkonen, M. : *Simultaneous estimation of internal admittivity distribution and electrode contact impedances*, Report 1 Univ. of Kuopio, 2002.

[2] Cheney, M., Isaacson, D. and Newwell, J., C. : *Electrical impedance tomography*, SIAM Rev., Vol. 41, No. 1, pp. 85-101, 1999.

[3] Michalewicz, Z.: *Genetic algorithms + Data Structure=Evolution Programs*. 2nd ed. Springer Verlag Berlin, 1994.

[4] Vauhkonen, M., Vadásy, D., Karjalainen, P. A., Somersalo, E. and Kaipio, J. P.: *Tikhonov regularization and prior information in electrical impedance tomography*, IEEE Trans. Med. Eng., Vol. 17, pp. 285-293, 1998.

[5] Borsic, A.: *Regularization methods for imaging from electrical measurement*, PhD. Thesis, Oxford Brookes University, 2002.

[6] Olmi, R., Bini, M. and Priori, S.: *A genetic algorithm approach to image reconstruction in electrical impedance tomography*, IEEE Trans. Evol. Comp., Vol. 4, pp. 83-88, 2000.

[7] Yorkey, T. J., Webster, J. G., Tomkins, W. J. : *Comparing reconstruction algorithms for electrical impedance tomography*, IEEE Trans. Biomed. Eng., Vol. 34, pp. 843-852, 1987.

[8] Bachorec, T., Jirku, T. and Dedkova, J. : *New Numerical Technique for Non-destructive Testing of the Conductive Materials,* in Proceedings of International Conference PIERS, Beijing, China, March 2007, pp. 977-981.

[9] Kubásek, R., Bachorec, T. and Dědková, J. : *Identification of Defects in Materials Using EIT*, in Electronic devices and systems, Brno, Czech Republic, September 2006, pp. 285-289.